GAMOW-TELLER STRENGTH DISTRIBUTIONS for $\beta\beta$-DECAYING NUCLEI WITHIN CONTINUUM-QRPA

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Abstract

A version of the pn-continuum-QRPA is outlined and applied to describe the Gamow-Teller strength distributions for $\beta\beta$-decaying open-shell nuclei. The calculation results obtained for the pairs of nuclei ^{116}Cd-Sn and ^{130}Te-Xe are compared with available experimental data.

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1 Introduction

Description of weak interaction in nuclei is often a challenge for models of nuclear structure. Numerous calculations of the nuclear $\beta\beta$-decay amplitudes well illustrate this statement (see, e.g. Refs. [1] and references therein). Uncertainties in theoretical calculations of the Gamow-Teller (GT) $2\nu\beta\beta$-decay amplitude $M^{2\nu}_{GT}$ have stimulated experimental studies of the GT$^{(\mp)}$-strengths of the 1^+ states virtually excited in the decay process (see, e.g. Refs. [2, 3]).

As a double charge-exchange process, $2\nu(0\nu)\beta\beta$-decay is enhanced by nucleon pairing which is due to the singlet part of the particle-particle (p-p) interaction. The discrete quasiboson version of the quasiparticle RPA (pn-dQRPA) which accounts for the nucleon pairing is usually applied to calculate the $\beta\beta$-decay amplitudes in open-shell nuclei [1]. In spite of differences in model parameterizations of the nuclear mean field and residual interaction in the particle-hole (p-h) and p-p channels, all pn-dQRPA calculations reveal marked sensitivity of the amplitude $M^{2\nu}_{GT}$ to the ratio g_{pp} of the triplet to singlet strength of the p-p interaction. Physical reasons for such a general feature of all calculations were analyzed in Ref. [4], where they were attributed to violation of the spin-isospin SU(4) symmetry in nuclei. An identity transformation of the amplitude into sum of two terms was used in Ref. [4]. One term, which is due to the p-p interaction only, depends linearly on g_{pp} and vanishes at $g_{pp} = 1$ when the SU(4)-symmetry is restored in the p-p sector of a model Hamiltonian. The second term is a smoother function of g_{pp} at $g_{pp} \sim 1$, but

exhibits a quadratic dependence on the strength of the mean-field spin-orbit term, which is the main source of violation of the spin-isospin SU(4)-symmetry in nuclei.

Understanding of general properties of the amplitude $M_{GT}^{2\nu}$ helps to improve reliability of evaluation of $\beta\beta$-decay amplitudes. For a quantitative analysis, we use here an isospin-selfconsistent pn-continuum-QRPA (pn-cQRPA) approach of Ref. [5], where this approach was applied to evaluate the $\mathrm{GT}^{(\mp)}$ strength distributions in single-open-shell nuclei. In the reference the full basis of the single-particle (s-p) states was used in the p-h channel along with the Landau-Migdal forces, while the nucleon pairing was described within the simplest version of the BCS-model based on discrete basis of s-p states. A rather old version of the phenomenological isoscalar nuclear mean field (including the spin-orbit term) was used in Refs. [5, 4], as well.

The first application of the pn-cQRPA approach of Ref. [5] to description of the $\beta\beta$-decay observables in several nuclei has been given recently in Ref. [7]. Realistic (zero-range) forces have been used in the p-p channel to describe the nucleon pairing within the BCS model realized on a rather large discrete+quasidiscrete s-p basis.

The pn-cQRPA approach of Refs. [5, 7] is further extended here by using a modern version of the phenomenological isoscalar mean field (including the spin-orbit term) deduced in Ref. [6] from the isospin-selfconsistent analysis of experimental single-quasiparticle spectra in double-closed-shell nuclei. In the present contribution we give a brief overview of the approach and its applications to description of different GT strength functions for the pairs of nuclei ^{116}Cd-Sn and ^{130}Te-Xe.

2 Coordinate representation of the pn-cQRPA equations and GT strength functions

In formulation of a version of the pn-cQRPA we follow Ref. [5], where the pn-dQRPA equations originally written in terms of the forward $X_s^{(-)}$ and backward amplitudes $Y_s^{(-)}$ are transformed in equivalent equations for the 4-component radial transition density $\rho_i^{(-)}(s,r)$. The latter is defined in terms of the X, Y amplitudes by Eqs. (39), (40) of Ref. [5] and related to the $\mathrm{GT}^{(-)}$ excitations having the wave functions $|1^+\mu, s\rangle$ and energies ω_s. The expression for the transition density $\rho_i^{(+)}(s,r)$ related to $\mathrm{GT}^{(+)}$ excitations follows from that for $\rho_i^{(-)}(s,r)$ by the substitution $p \leftrightarrow n$. Hereafter, we often use the notations of Ref. [5] (except for the energies) and refer to some equations from this reference. Limiting ourselves in this contribution to description of the GT transitions only, we omit the quantum numbers indices $J = S = 1$, $L = 0$ for spin-monopole excitations. The spin-angular variables in all expressions are separated out as well.

The pn-dQRPA solutions ω_s are related to the excitation energies $E_{x,s}^{(\mp)}$ measured from the ground-state energy $E_0(Z \pm 1, N \mp 1)$ of the corresponding daughter nuclei as:

$$\omega_s \pm (\mu_p - \mu_n) = \omega_s^{(\mp)} = E_{x,s}^{(\mp)} + Q_b^{(\mp)}. \tag{1}$$

Here, $\mu_{p(n)}$ is the chemical potential for the proton (neutron) subsystem found from the known BCS equations, $Q_b^{(\mp)} = \mathcal{E}_b(Z,N) - \mathcal{E}_b(Z \pm 1, N \mp 1)$ are the total binding-energy differences, $\omega_s^{(\mp)}$ are the excitation energies measured from $E_0(Z,N) - \sum_a m_a = -\mathcal{E}_b(Z,N)$ (m_a is the nucleon mass). The energies $\omega_s^{(\mp)}$ are usually described by a model Hamiltonian.

The system of equations for $\rho_i^{(-)}(s,r)$ (Eq.(41) of Ref. [5]) contains the explicit expression for the 4×4 matrix of the free two-quasiparticle propagator $A_{ik}^{(-)}(r,r',\omega_s)$ (Eq.(43) of Ref. [5]; $A_{ik}^{(+)} = A_{ik}^{(-)}(p\leftrightarrow n)$). These propagators are the main quantities in description of charge-exchange excitations within the pn-QRPA. In particular, in terms of A_{ik} one can formulate a Bethe-Salpeter-type equation for the effective propagator $\tilde{A}_{ik}(r,r',\omega_s)$ [7]. The spectral expansion of $\tilde{A}_{ik}$ in terms of $\rho_i(s,r)$ allows one to express the pn-QRPA strength functions in terms of the effective propagator, or, equivalently, in terms of the 4-component effective fields [5]. Some relevant formulas are shown below.

The $\mathrm{GT}^{(\mp)}$ strength functions, corresponding to the external fields (probing operators) $\hat{V}_\mu^{(\mp)} = \sum_a V_\mu^{(\mp)}(a)$, $V_\mu^{(\mp)} = \sigma_\mu \tau^{(\mp)}$, are defined as follows:

$$S^{(\mp)}(\omega) = \sum_s |\langle 1^+, s\| \hat{V}^{(\mp)} \|0\rangle|^2 \delta(\omega - \omega_s) \qquad (2)$$

with GT strengths $B_s^{(\mp)}(GT) = |\langle 1^+, s\|\hat{V}^{(\mp)}\|0\rangle|^2$. The strength function $S^{(-)}(\omega)$ can be expressed in terms of the corresponding effective field $\tilde{V}_{i[1]}^{(-)}(r,\omega)$, which is different from the external one $V_i^{(-)}(r) = \delta_{i1}$ due to the residual interaction [5]:

$$S^{(-)}(\omega) = -\frac{3}{\pi} Im \sum_i \int A_{1i}^{(-)}(r,r',\omega)\tilde{V}_{i[1]}^{(-)}(r',\omega)drdr', \qquad (3)$$

$$\tilde{V}_{i[1]}^{(-)}(r,\omega) = \delta_{i1} + \frac{F_i^{(1)}}{4\pi r^2}\sum_k \int A_{ik}^{(-)}(r,r',\omega)\tilde{V}_{i[1]}^{(-)}(r',\omega)dr'. \qquad (4)$$

The residual interaction here is supposed to be of zero-range type with intensities $F_i^{(1)}$: $F_1^{(1)} = F_2^{(1)} = 2G'$, $F_3^{(1)} = F_4^{(1)} = G_1$. For the 0^+ p-h and p-p channels the corresponding strengths are: $F_1^{(0)} = F_2^{(0)} = 2F'$ and $F_3^{(0)} = F_4^{(0)} = G_0$, respectively. Dimensionless values $g' = G'/C$, $f' = F'/C$, ($C = 300\ MeV \cdot fm^3$) are the well-known Landau-Migdal p-h strength parameters. The same parameterization we use for the p-p interaction strengths: $g_1 = G_1/C$, $g_0 = G_0/C$. For calculation of $S^{(+)}(\omega)$ one can use Eqs. (3), (4) with substitution $p\leftrightarrow n$ [5]. An alternative way is based on the symmetry properties of A_{ik}: $A_{11}^{(+)} = A_{22}^{(-)}$. As a result, we get the expression for $S^{(+)}(\omega)$ in terms of $A_{ik}^{(-)}$ and $\tilde{V}_i^{(-)}$. This expression is obtained from Eqs. (3), (4) with the substitution $1\rightarrow 2$.

The nuclear $\mathrm{GT}^{(-)}$ amplitude for $2\nu\beta\beta$-decay into the ground state $|0'\rangle$ of the product nucleus $(N-2, Z+2)$ is given by the expression:

$$M_{GT}^{2\nu} = \sum_s \frac{\langle 0'\|\hat{V}^{(-)}\|1^+,s\rangle\langle 1^+,s\|\hat{V}^{(-)}\|0\rangle}{\bar{\omega}_s}, \qquad (5)$$

where $\bar{\omega}_s = E_s - \frac{1}{2}(E_0 + E_{0'}) = E_{x,s} + \frac{1}{2}(Q_b^{(-)} + Q_b^{(+)'})$. To calculate $M_{GT}^{2\nu}$ within the pn-QRPA, the vacua $|0\rangle$ and $|0'\rangle$ should be identified. As a result of such identification, one has $\bar{\omega}_s = \frac{1}{2}(\omega_s^{(-)} + \omega_s^{(+)'}) \approx \omega_s$, in accordance with Eq. (1).

The amplitude (5) can be expressed in terms of a "non-diagonal" $\mathrm{GT}^{(-)}$ strength function $S^{(--)}(\omega)$:

$$M_{GT}^{2\nu} = \int \omega^{-1} S^{(--)}(\omega)d\omega, \qquad (6)$$

where $S^{(--)}(\omega)$ is defined as follows:

$$S^{(--)}(\omega) = \sum_s \langle 0'\|\hat{V}^{(-)}\|1^+,s\rangle\langle 1^+,s\|\hat{V}^{(-)}\|0\rangle\delta(\omega - \bar{\omega}_s). \qquad (7)$$

The corresponding pn-QRPA expression for $S^{(--)}$ is:

$$S^{(--)}(\omega) = -\frac{3}{\pi} Im \sum_i \int A_{2i}^{(-)}(r, r', \omega) \tilde{V}_{i[1]}^{(-)}(r', \omega) dr dr'. \tag{8}$$

An alternative expression for $M_{GT}^{2\nu}$ is obtained in terms of the "non-diagonal" static polarizibility [7]:

$$M_{GT}^{2\nu} = -\frac{3}{2} \sum_i \int A_{2i}^{(-)}(r, r', \omega = 0) \tilde{V}_{i[1]}^{(-)}(r', \omega = 0) dr dr'. \tag{9}$$

Decomposition of the amplitude (5) into two terms [4]

$$M_{GT}^{2\nu} = (M_{GT}^{2\nu})' + \bar{\omega}_{GTR}^{-2} EWSR^{(--)}, \tag{10}$$

$$EWSR^{(--)} = \sum_s \bar{\omega}_s \langle 0' \| \hat{V}^{(-)} \| 1^+, s \rangle \langle 1^+, s \| \hat{V}^{(-)} \| 0 \rangle, \tag{11}$$

where $\bar{\omega}_{GTR}$ is the energy of GT$^{(-)}$ giant resonance (GTR), allows us to clarify the sensitive dependence of $2\nu\beta\beta$-decay amplitude as a function of g_{pp} (for details, see Ref. [4]). The "non-diagonal" energy-weighted sum rule $EWSR^{(--)}$ is straightforwardly expressed in terms of the strength function $S^{(--)}$ of Eq. (6):

$$EWSR^{(--)} = \int \omega S^{(--)}(\omega) d\omega, \tag{12}$$

again supposing the QRPA vacuum $|0'\rangle$ is identified with that of $|0\rangle$.

3 Calculation of strength function within the pn-cQRPA

Starting from the coordinate representation of the pn-dQRPA equations outlined above, we are able to take exactly into account the s-p continuum in the p-h channel and, therefore, to formulate a version of the pn-cQRPA. The pairing problem is solved on a rather large basis of bound+quasibound proton and neutron s-p states within the present version of the model. To take the s-p continuum into account, the following transformations of the expression for $A_{ik}(r, r', \omega)$ [5] are done: (i) the Bogolyubov coefficients v_λ, u_λ and the quasi-particle energies E_λ are approximated by their non-pairing values $v_\lambda = 0$, $u_\lambda = 1$, and $E_\lambda = \varepsilon_\lambda - \mu$ for those s-p states (λ), which lie far above the chemical potential (i.e. $\varepsilon_\lambda - \mu \gg \Delta_\lambda$), (ii) the Green function of the s-p radial Schrödinger equation $g_{(\lambda)}(r, r', \varepsilon) = \sum_{\varepsilon_\lambda} (\varepsilon - \varepsilon_\lambda + i0)^{-1} \chi_\lambda(r) \chi_\lambda(r')$, which is calculated via the regular and irregular solutions of this equation, is used to perform explicitly the sum over the s-p states in the continuum. As a result, the properly transformed free two-quasiparticle propagator A is obtained, upon which a corresponding version of the pn-cQRPA is based.

The solution of the pairing problem is simplified by using the "diagonal" approximation for the p-p interaction for the 0^+ neutral channel. In this approximation the nucleon-pair operators are assumed to be formed only from the pair of nucleons occupying the same s-p level λ. The nucleon pairing is described with the use of the Bogolybov transformation with the gap parameter Δ_λ dependent on λ. The same number N_{b+qb} of bound+quasibound states forming the basis of the BCS problem is used for both the neutron and proton subsystems. These numbers are shown in Table 1 for nuclei in question.

In evaluation of total binding energies within the model (that is necessary to evaluate the pairing energies $\mathcal{E}_{pair}$) the blocking effect for odd nuclei is taken into account. In description of the nucleon pairing, different values of the p-p interaction strength parameters $g_{0,n}$ and $g_{0,p}$ for the neutron and proton subsystems are used. These values are found from comparison of the calculated and experimental pairing energies for nuclei under consideration (Table 1).

Table 1: The phenomenological mean field parameters (U_0, U_{SO} and a), singlet and triplet p-h and p-p interaction strengths (f', g', g_0, g_{pp}) used in calculations.

Pair of nuclei	U_0, MeV	U_{SO}, MeV·fm^2	a, fm	f'	$g_{0,n}$	$g_{0,p}$	N_{b+qb}	g'	g_{pp}
^{116}Cd-^{116}Sn	51.62	34.08	0.618	1.06	0.388	0.333	22	0.77	1.0
^{130}Te-^{130}Xe	51.74	34.025	0.628	1.09	0.356	0.364	22	0.88	0.99

The mean field consists of the phenomenological isoscalar part (including the spin-orbit term) along with the isovector and Coulomb part (Eq. (1) of Ref. [5]). The parameterization of the Woods-Saxon-type isoscalar part contains two strength (U_0, U_{SO}) and two geometrical (r_0, a) parameters [6]. The mean field isovector part (the symmetry potential) is calculated in an isospin-selfconsistent way (Eqs. (7), (35) of Ref. [5]) via the neutron-excess density and Landau-Migdal strength parameter f'. The mean Coulomb field is also calculated selfconsistently via the proton density. All densities are calculated with taking into account the nucleon pairing. Five above-listed model parameters found in Ref. [6] for a number of double-closed-shell nuclei are properly interpolated for nuclei under consideration (see Table 1; $r_0 = 1.27$ fm is taken for all nuclei).

The values of the Landau-Migdal strength g' listed in Table 1 are obtained by fitting the experimental GTR energy in calculations of the GT$^{(-)}$ strength function. The p-p interaction strength g_1 (or its relative value $g_{pp} = 2g_1/(g_{0,n} + g_{0,p})$) is considered as a free parameter. It can be adjusted to reproduce the experimental $M^{2\nu}_{GT}$ value (the corresponding values are listed in the last column of Table 1).

Considering the pair ^{116}Cd-^{116}Sn, the GT$^{(-)}$ strength distribution calculated within the pn-dQRPA for the transition ^{116}Sn$\rightarrow$^{116}Sb is shown in Fig. 1a (a small imaginary part is added to the s-p potential). To compare the calculation results with the ^{116}Sn(^{3}He,t) experimental data of Ref. [8], five centroids of the energy, $E_{x,i}$, and their strength x_i relative to the one of the GTR are evaluated (Fig. 1b). The value $g' = 0.77$ allows to reproduce the experimental GTR energy in the calculation. The GT$^{(-)}$ strength distribution is almost insensitive to the g_{pp} value ($g_{pp} = 1.0$ is taken in the calculation). The GT$^{(+)}$ strength distribution for the transition ^{116}Sn$\rightarrow$^{116}In is found more sensitive to g_{pp}. Only one 1^+ state with $B^{(+)}(GT) = 0.47$ corresponding to the $1g^p_{9/2} \rightarrow 1g^n_{7/2}$ transition into the ^{116}In ground state, is found in the calculation within the interval $E_x < 5\ MeV$. This weak transition is allowed due to the neutron pairing in ^{116}Sn. In the ^{116}Sn(d,^{2}He) experiments four 1^+ states in ^{116}In were found within the interval $E_x \leq 3\ MeV$ with total strength $\sum_i B^{(+)}_i(GT) = 0.66$ [3]. Population of the 1^+ states in ^{116}In has also been studied in the ^{116}Cd(p,n)-reaction [2]. The result $B^{(-)}(GT) = 0.26 \pm 0.02$ for excitation of the ^{116}In ground state is only available now. Within the interval $E_x \leq 3\ MeV$ the calculated GT$^{(-)}$ strength distribution in ^{116}In exhibits one 1^+ state, corresponding to the back-spin-flip transition $1g^n_{7/2} \rightarrow 1g^p_{9/2}$ into the ^{116}In ground state with the value 1.05 $B^{(-)}(GT)$. (for ^{116}Sn-^{116}Sb this transition is Pauli blocked). The $2\nu\beta\beta$-decay amplitude

for the decay ^{116}Cd$\rightarrow^{116}$Sn can barely be evaluated within the pn-QRPA because the proton shell is closed in ^{116}Sn.

Coming to the pair ^{130}Te-^{130}Xe, the value $g' = 0.88$ is found in the calculation by fitting the experimental GTR energy in ^{130}I [9]. Then the amplitude $M^{2\nu}_{GT}$ (6) (or (9)) and its decomposition (10) are calculated, as a function of g_{pp} (Fig. 2). The corresponding experimental value $(M^{2\nu}_{GT})^{exp} = 0.03$ MeV^{-1} [10] can be reproduced in the calculation at $g_{pp} = 0.99$. The $2\nu\beta\beta$-decay strength function $\omega^{-1}S^{(--)}(\omega)$ is calculated for this value of g_{pp} (Fig. 3a) along with the corresponding running sum $M^{2\nu}_{GT}(\omega) = \int^{\omega} \omega'^{-1}S^{(--)}(\omega')d\omega'$ (Fig. 3b). Figs. 2 and 3 illustrate how the $M^{2\nu}_{GT}$ value for the decay ^{130}Te$\rightarrow^{130}$Xe is formed. In particular, as one sees in Fig.3, the experimental studies of $B^{(+)}(GT)$ are not always sufficient for understanding partial contributions to $M^{2\nu}_{GT}$. The reason is that the intermediate states having a relatively large excitation energy and very small $B^{(+)}(GT)$ value (like the GTR) can nonetheless play essential role in formation of the $2\nu\beta\beta$-decay amplitude.

In conclusion, an isospin-selfconsistent version of the pn-cQRPA has been outlined and some its applications to description of charge-exchange excitations in open-shell spherical nuclei are presented. Although only general features of the low-energy strength distributions can be described within the approach, it seems applicable to analysis of $\beta\beta$-decay observables.

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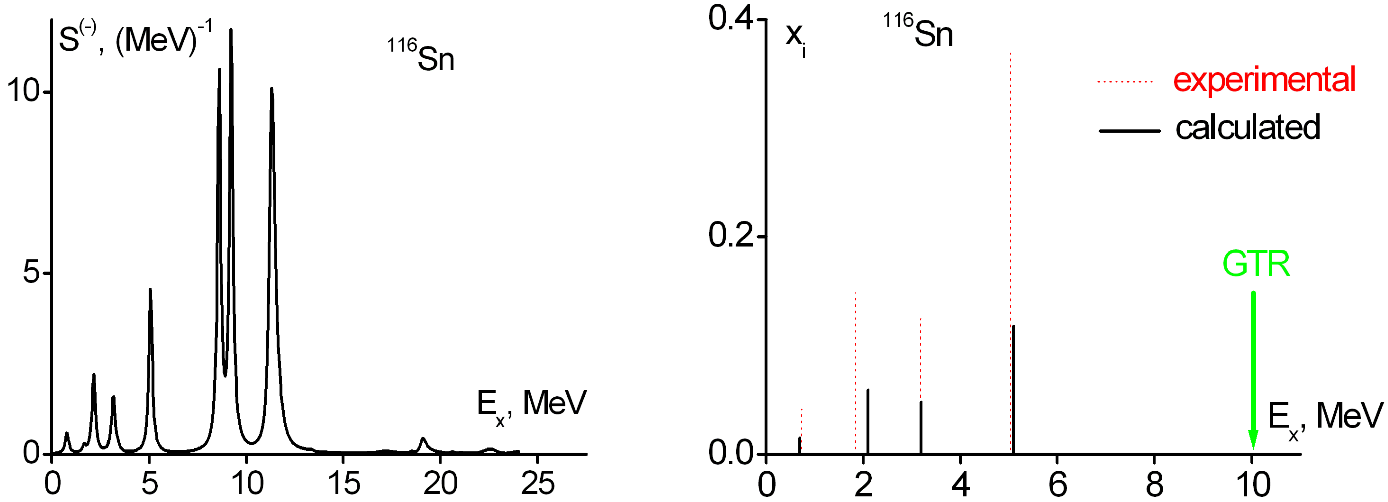

Figure 1: The GT$^{(-)}$ strength function for ^{116}Sn-^{116}Sb (a) and the relative (with respect to the GTR) strength of the low-energy 1^+ peaks calculated within pn-cQRPA (b). The corresponding experimental data are taken from Ref. [8]

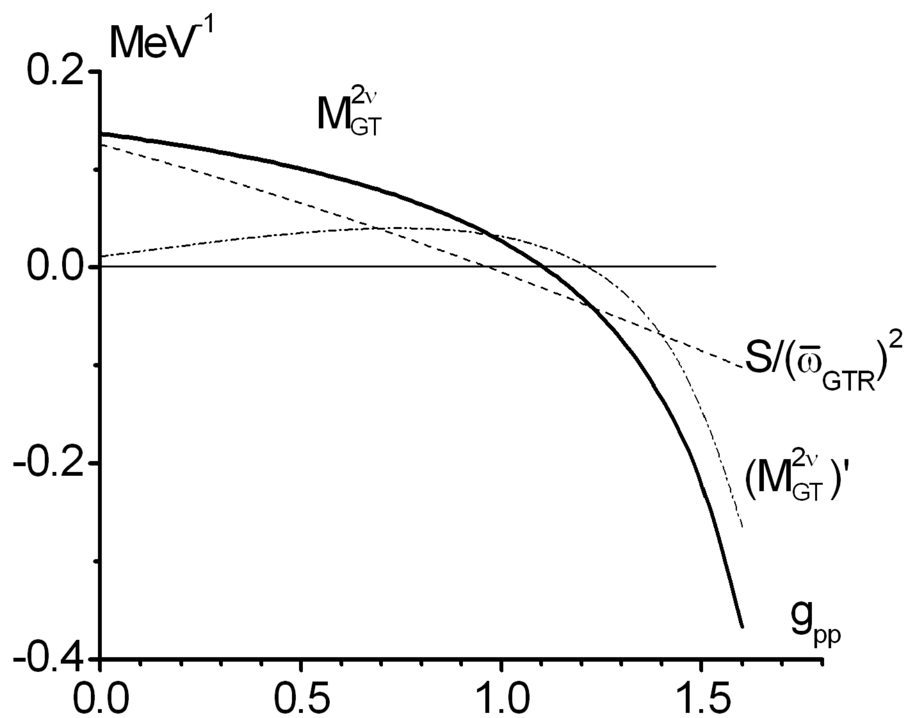

Figure 2: The calculated amplitude of ^{130}Te $2\nu\beta\beta$-decay as a function of g_{pp}. Decomposition of Eq. (10) is also shown.

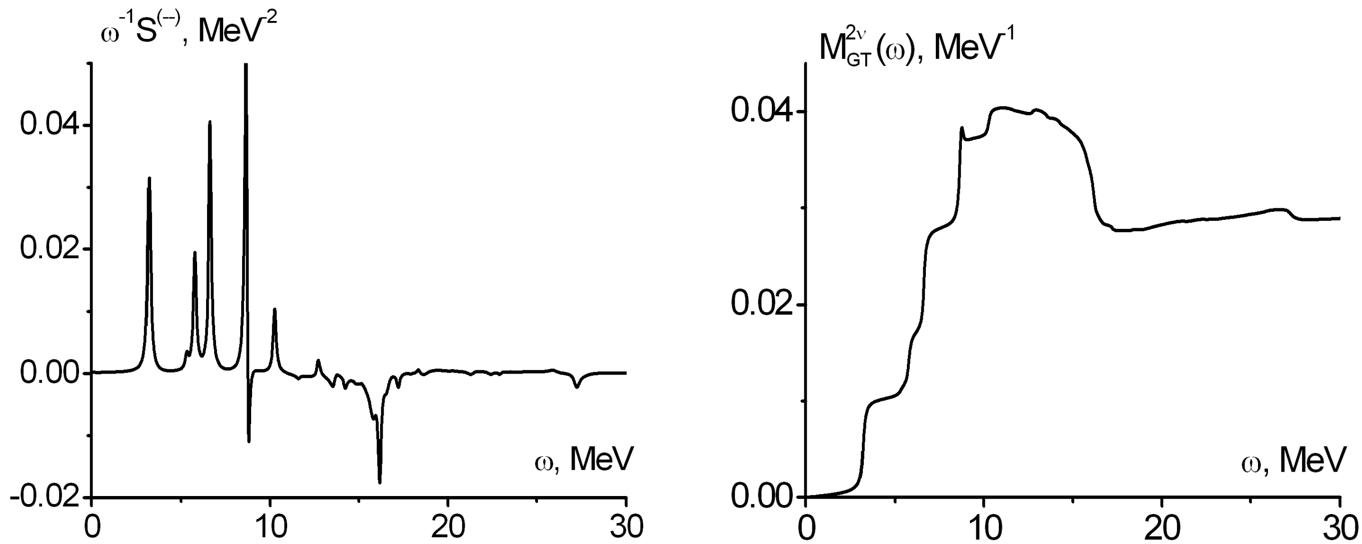

Figure 3: The GT $2\nu\beta\beta$-decay strength function (a) and the running sum (b) calculated for ^{130}Te at $g_{pp} = 0.99$